SEMINARIO

Mario Richiutto

INRIA at University of Bordeaux

Steady state preservation, well balanced and error balance for hyperbolic equations: from consistency with constants to the preservation of differential operators.

Abstract: Even in one space dimension, the numerical approximation of hyperbolic conservation laws is a challenging task due to the complex structure of the Riemann problem introducing solutions composed of smooth regions separated by discontinuities. Steady states of hyperbolic conservation laws, however, involve either constant solutions, or constant solutions separated by isolated discontinuities (shocks or contact discontinuities). When an additional forcing term is included in the PDE, one speaks of hyperbolic balance laws. In this case, the structure of the solutions becomes even richer. Indeed, due to the presence of the additional source term, the PDE may exhibit families of non-trivial steady equilibria, which may involve complex variations of the variables as well as discontinuities. Many such states have physical relevance, and in several applications, one is confronted with the need of studying the evolution of very small perturbations of one of such equilibrium states is necessary. Examples can be found both in one and several space dimensions in coastal engineering (e.g. tsunamis, bore propagation in rivers), aerospace and propulsion (e.g. small perturbations of stationary flows in nozzles), astrophysics and large scale geophysical (e.g. small perturbations of vortical flows with gravitational effects). From the numerical point of view, the approximation of constants is the usual, most natural consistency condition. This means that any perturbation of a constant state can be easily studied numerically, with some classical accuracy and resolution constraints. Exact consistency with non-constant states is, however, not trivial. This has motivated over the last roughly 40 or 50 years the research on numerical methods capable of preserving certain steady state equilibria. This property has been referred to as C-property, of well balanced, or quite simply ‘steady state preserving’ property. In this short course, we will review the issue of preserving steady states in one and multiple space dimensions. We will show how this notion can be linked to geometrical, and implicit high order ODE integrations in one space dimension. The extension to multiple space dimensions naturally leads to the notion of compatible spatial approximations, capable of preserving differential constraints. Some examples of numerical methods and some numerical results are used to illustrate the theoretical notions.

Sala de Grados I, Facultad de Ciencias
Miércoles 17 de Septiembre de 2025 (16:00)
Organiza: G.I.R. MTANPOEE